Chapter 8: Application of integral in real life

1 Probability

1.1 PDF and CDF

Definition 1.1. A function $p(x)$ is a **probability density function** or PDF if it satisfies the following conditions

- $p(x) \geq 0$ for all x.
- $\int_{-\infty}^{\infty} p(x) = 1$.

Definition 1.2. A function $P(t)$ is a **Cumulative Distribution Function** or cdf, of a density function $p(t)$, is defined by

$$P(t) = \int_{-\infty}^{t} p(x)dx$$

Which means that $P(t)$ is the antiderivative of $p(t)$ with the following properties:

- $P(t)$ is increasing and $0 \leq P(t) \leq 1$ for all t.
- $\lim_{t\to\infty} P(t) = 1$.
- $\lim_{t\to-\infty} P(t) = 0$.

Moreover, we have $\int_a^b p(x)dx = P(b) - P(a)$.

1.2 Probability, mean and median

Probability

Let us denote X to be the quantity of outcome that we care (X is in fact, called the random variable).

$$\mathbb{P}\{a \leq X \leq b\} = \int_a^b p(x)dx = P(b) - P(a)$$

$$\mathbb{P}\{X \leq t\} = \int_{-\infty}^{t} p(x)dx = P(t)$$

$$\mathbb{P}\{X \geq s\} = \int_s^{\infty} p(x)dx = 1 - P(s)$$

The mean and median

Definition 1.3. A **median** of a quantity X is a value T such that the probability of $X \leq T$ is $1/2$. Thus we have T is defined by the value such that

$$\int_{-\infty}^{T} p(x)dx = 1/2$$

or

$$P(T) = 1/2$$

.

Definition 1.4. A **mean** of a quantity X is the value given by

$$Mean = \frac{\text{Probability of all possible quantity}}{\text{Total probability}} = \frac{\int_{-\infty}^{\infty} xp(x)dx}{\int_{-\infty}^{\infty} p(x)dx} = \frac{\int_{-\infty}^{\infty} xp(x)dx}{1} = \int_{-\infty}^{\infty} xp(x)dx.$$

Normal Distribution

Definition 1.5. A normal distribution has a density function of the form

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean of the distribution and σ is the standard deviation, with $\sigma > 0$. The case $\mu = 0$, $\sigma = 1$ is called the standard normal distribution.