1. Handout Solutions

1.1. **(1).** This is a partial fraction problem.

Set the partial fraction for

$$\begin{aligned}&\frac{x^2-8x+3}{(x-2)(x+1)^2}\\=&\frac{A}{x-2}+\frac{B}{(x+1)^2}+\frac{C}{x+1}\\=&\frac{Ax^2+2Ax+A+Bx-2B+Cx^2-Cx-2C}{(x-2)(x+1)^2}\\=&\frac{(A+C)x^2+(2A+B-C)x+A-2B-2C}{(x-2)(x+1)^2}\end{aligned}$$

Thus we have $\begin{cases}A+C=1\\2A+B-C=-8\\A-2B-2C=3\end{cases}$

Solve it, we have $\begin{cases}A=-1\\B=-4\\C=2\end{cases}$

Therefore, we have

$$\int\frac{x^2-8x+3}{2(x-2)(x+1)^2}dx=\frac{1}{2}\int\frac{-1}{x-2}+\frac{-4}{(x+1)^2}+\frac{2}{x+1}dx=-\frac{1}{2}\ln|x-2|+\ln|x+1|+\frac{2}{x+1}+C$$

1.2. **(2).** This is a partial fraction problem.

Set the partial fraction for

$$\begin{aligned}&\frac{6x^2-8x+11}{(2x+1)(2x^2+5)}\\=&\frac{A}{2x+1}+\frac{Bx+C}{2x^2+5}\\=&\frac{2Ax^2+5A+2Bx^2+Bx+2Cx+C}{(2x+1)(2x^2+5)}\\=&\frac{(2A+2B)x^2+(B+2C)x+5A+C}{(2x+1)(2x^2+5)}\end{aligned}$$

Thus we have $\begin{cases}2A+2B=6\\B+2C=-8\\5A+C=11\end{cases}$

Solve it, we have $\begin{cases}A=3\\B=0\\C=-4\end{cases}$

Therefore, we have

$$\int\frac{6x^2-8x+11}{(2x+1)(2x^2+5)}dx=\int\frac{3}{2x+1}+\frac{-4}{2x^2+5}dx=\frac{3}{2}\ln|2x+1|-\frac{2\sqrt{2}}{\sqrt{5}}\arctan\sqrt{\frac{2}{5}}x+C$$

1.3. **(3).** This is a partial fraction problem.

Set the partial fraction for

$$\begin{aligned}&\frac{2x^3+4x^2+3x+11}{(x+1)^2(x^2+4)}\\=&\frac{A}{(x+1)^2}+\frac{B}{x+1}+\frac{Cx+D}{x^2+4}\\=&\frac{Ax^2+4A+Bx^3+Bx^2+4Bx+4B+Cx^3+2Cx^2+Cx+Dx^2+2Dx+D}{(x+1)^2(x^2+4)}\\=&\frac{(B+C)x^3+(A+B+2C+D)x^2+(4B+2D+C)x+4A+4B+D}{(x+1)^2(x^2+4)}\end{aligned}$$

Thus we have $\begin{cases}B+C=2\\A+B+2C+D=4\\4B+2D+C=3\\4A+4B+D=11\end{cases}$

Solve it, we have $\begin{cases}A=2\\B=1\\C=1\\D=-1\end{cases}$

Therefore, we have

$$\int\frac{2x^3+4x^2+3x+11}{(x+1)^2(x^2+4)}dx=\int\frac{1}{(x+1)^2}+\frac{1}{x+1}+\frac{x-1}{x^2+4}dx=\frac{1}{2}\ln|x^2+4|-\frac{2}{x+1}+\ln|x+1|+\frac{1}{2}\arctan\frac{x}{2}+C$$

1.4. **(4).** It looks like that we should do trig substitution for this one.... But wait!

We can actually do this with normal substitution!

So you can take $u=2-3x^2$ and thus $du=-6xdx$. Then the original integral becomes

$$\int\frac{x}{\sqrt{2-3x^2}}dx=-\int\frac{1}{6\sqrt{u}}du=-\frac{1}{3}\sqrt{2-3x^2}+C$$

1.5. **(5).** For this one, unfortunately, we have to use Trigonometric substitution.

Since we are in the scenario with terms $A-Bx^2$ involved with $A=2$ and $B=3$, we can take $x=\sqrt{\frac{2}{3}}\sin(u)$. Now the original integration becomes

$$\int\frac{1}{\sqrt{2-3x^2}}dx=\int\frac{1}{\sqrt{3}}du=\frac{1}{\sqrt{3}}u+C$$

Now plug it back in $u=\arcsin(\sqrt{\frac{3}{2}}x)$, we have the integral equals $\frac{1}{\sqrt{3}}\arcsin(\sqrt{\frac{3}{2}}x)+C$.